# SWCNT/Al<sub>2</sub>O<sub>3</sub> Hybrid Nanofluid flow on a Stagnation Point with Thermal Radiation Effects

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ABSTRACT: In this study, the flow and heat transfer for SWCNT-Al<sub>2</sub>O<sub>3</sub>/Water hybrid nanofluid on a stagnation point with thermal radiation effects and Newtonian heating are numerically examined. The transformed ordinary differential was solved numerically using the Runge-Kutta-Fehlberg (RKF45) method. Fluid flow characteristics in term of variation of surface temperature and skin friction coefficient towards changes of the nanoparticle volume fraction, the thermal radiation parameter and the conjugate parameter are analysed and discussed.

**Keywords:** Carbon nanotube; Hybrid nanofluid; Stagnation point flow.

# 1. INTRODUCTION

It is known that the nanofluid blend from the metal nanoparticles provided better performance in heat transfer capability compared to oxide nanoparticle [1]. But, metal nanoparticle like silver Ag is expensive and denser, thus contribute to erosion on the component surface. The evolution study on nanofluid found that the use of carbon nanotube like SWCNT desired the performance provided by the silver while lowering the friction. Unfortunately, the carbon nanotube is still expensive. The present study proposed cheaper hybrid nanofluid by blending together the cheap oxide nanoparticles alumina  $Al_2O_3$  with SWCNT in water hybrid nanofluid. The thermal radiation effect is taking account as it provided sustainable and renewable energy.

From the literature study, this kind of study is never been held before, thus, the reported results in this study are new.

## 2. MATHEMATICAL FORMULATION

A steady two-dimensional stagnation point flow towards a flat plate immersed in SWCNT-Al<sub>2</sub>O<sub>3</sub>/Water hybrid nanofluid with ambient temperature,  $T_{\infty}$  is considered. u and v are assumes as the velocity components along the x and y axes, respectively. The free stream velocity  $U_{\infty} = ax$  are assumed in linear forms where a is a positive constant and  $q_r$  is the radiative

heat flux. Subjected to the Newtonian heating boundary conditions, the governing equations for this heat transfer and fluid flow model can be expressed as [2, 3]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_{\infty}\frac{dU_{\infty}}{dx} + v_{hnf}\frac{\partial^2 u}{\partial y^2},$$
 (2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\left(\rho C_p\right)_{hnf}} \frac{\partial q_r}{\partial y}, \quad (3)$$

with boundary conditions

$$u = 0, \ v = 0, \ \frac{\partial T}{\partial y} = -h_s T \text{ at } y = 0,$$
  
 $u \to U_{\infty}, \quad T \to T_{\infty}, \text{ as } y \to \infty.$  (4)

The hybrid nanofluid kinematic viscosity, dynamic viscosity and its density is denoted as  $V_{hnf}$ ,  $\mu_{hnf}$  and  $\rho_{hnf}$ , respectively. T is the temperature inside the boundary layer,  $(\rho C_p)_{hnf}$  is the heat capacity of hybrid nanofluid and  $k_{hnf}$  is the thermal conductivity of hybrid nanofluid. Other properties related to base fluid and the nanoparticles are denoted in [3].

Next, substitute the similarity variables in [3, 4] into governing equations (2) and (3) gives the following transformed ordinary differential equations:

$$\frac{v_{hnf}}{v_f} f''' + ff'' - f'^2 + 1 = 0$$
 (5)

$$\frac{k_{hnf}(\rho C_p)_f}{k_f(\rho C_n)_{hnf}} \frac{1}{\Pr} \left( 1 + \frac{4}{3} N_r \right) \theta'' + f \theta' = 0.$$
 (6)

The boundary conditions becomes

$$f(0) = 0, \ f'(0) = 0, \ \theta'(0) = -\gamma (1 + \theta(0)),$$
  
 $f'(\eta) \to 1, \ \theta(\eta) \to 0, \text{ as } y \to \infty.$  (7)

The physical quantities interested are the surface temperature  $\theta(0)$ , the heat transfer rate  $-\theta'(0)$  and the skin friction coefficient  $C_f$  which can be reduced

$$C_f \operatorname{Re}_x^{1/2} = \frac{f''(0)}{(1-\phi_1)^{2.5} (1-\phi_2)^{2.5}},$$
 (8)

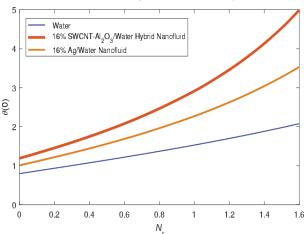
where  $\operatorname{Re}_{x} = \frac{U_{\infty}x}{v_{f}}$  is the Reynolds number.

## 3. RESULTS AND DISCUSSION

The system of ordinary differential equations (5) and (6) with boundary conditions (7) were solved numerically using the Runge-Kutta-Fehlberg (RKF45) technique. The numerical results obtained for the surface temperature  $\theta(0)$ , the heat transfer rate  $-\theta'(0)$  and the reduced skin friction coefficient  $C_f \operatorname{Re}_x^{1/2}$  for a various values of conjugate parameter  $\gamma$ , the thermal radiation parameter  $N_r$  and the nanoparticle volume fraction for alumina  $\operatorname{Al}_2\operatorname{O}_3(\phi_1)$  and SWCNT  $(\phi_2)$ .

Figure 1 shows the variation of the surface temperature  $\theta(0)$  for various values of the thermal radiation parameter  $N_r$ . From this figure, it is observed that the values of  $\theta(0)$  is increases as the values of  $N_r$  increases. Logically, the pattern of variation for the heat transfer rate  $-\theta'(0)$  is the same as  $\theta(0)$ , due to the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature Further, it is found that the present of nanoparticles in water based-fluid enhanced the values of  $\theta(0)$ . Figure 1 proposed that the SWCNT-Al2O3/Water hybrid nanofluid provided the highest values of  $\theta(0)$  compared to water and Ag/Water nanofluid.

Figure 1 Variation of the surface temperature  $\theta(0)$  for various values of  $N_r$  when Pr = 6.2,  $\gamma = 0.5$ 



In considering the fluid friction of the fluid with the plate surface, Table 1 shows that the water based-fluid provided the least friction due to none nanoparticles content. As the nanoparticles presence, the fluid friction increases. From Table 1, the nanofluid Ag/Water has the highest values of  $C_f \operatorname{Re}_x^{1/2}$ , physically contributed by the Ag density. Blending the Al<sub>2</sub>O<sub>3</sub> and SWCNT together in a fluid have results the lowest  $C_f \operatorname{Re}_x^{1/2}$  among nanofluid

tested.

Table 1 Values of the reduced skin friction coefficient  $C_r$  Re<sub>x</sub><sup>1/2</sup> for various fluid when Pr = 6.2,  $\gamma = N_r = 0.5$ 

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Fluid	$C_f \operatorname{Re}_x^{1/2}$
Water based-fluid	1.23259
16% Al <sub>2</sub> O <sub>3</sub> / Water nanofluid	1.86283
16% Ag/ Water nanofluid.	2.43558
16% SWCNT-Al <sub>2</sub> O <sub>3</sub> /Water hybrid nanofluid	1.78252

#### 4. CONCLUSION

In a conclusion, the hybrid nanofluid proposed in this study shows the win solution in term of performance, reliability and economy. The production cost of this kind of nanofluid is lower compared to metal nanofluid but provided better in term of heat transfer ability as well as reduced friction which prolong the component surface. Furthermore, the presence of thermal radiation has increased the surface temperature and the heat transfer rate but gave no effect on the skin friction coefficient.

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